Interpretation of a confidence interval

"We are 95% confident that μ lies between _____ and ____."

What that does really mean?

95% means — If we repeat our study 100 times,

- roughly 95 sample means stay **within** the $\pm 2\frac{\sigma}{\sqrt{n}}$ neighborhood of μ
- roughly 5 sample means stay **outside** the $\pm 2\frac{\sigma}{\sqrt{n}}$ neighborhood of μ

*"We are hoping that our $\bar{x}$ is one of the 95 "**lucky**" sample means that stays close to μ. If so, then our estimate of μ will be **accurate** i.e. μ lies within our confidence interval.*

*If our $\bar{x}$ is one of the 5 "**unlucky**" sample means that stays far from μ, then our estimate of μ will be **inaccurate** i.e. μ lies outside our confidence interval.*

*We will **never know** if our $\bar{x}$ is the lucky 95% or unlucky 5%."*

Think about your confidence level as a success rate of estimating μ. For example, a 80% confidence level means that your estimate is correct 80% of the time. Of course, a higher success rate it always better but it comes with a price! We will explore that in our homework.

bad $\bar{x}$'s $\bar{x}$ ← good $\bar{x}$'s   bad $\bar{x}$'s

$-2\frac{\sigma}{\sqrt{n}}$ μ $+2\frac{\sigma}{\sqrt{n}}$

μ captured!

Mission accomplished!