Math 101 Ch 2

2

Describing Distributions with Numbers

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In this chapter, we cover...

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In this chapter, we cover...

2.1 Measuring center: The mean
2.2 Measuring center: The median
2.3 Comparing the mean and the median
2.4 Measuring variability: The quartiles
2.5 The five-number summary and boxplots
2.6 Spotting suspected outliers and modified boxplots*
2.7 Measuring variability: The standard deviation
2.8 Choosing measures of center and variability
2.9 Examples of technology
2.10 Organizing a statistical problem

We saw in Chapter 1 (page 14) that the American Community Survey asks, among much else, workers' travel times to work. Here are the travel times in minutes for 15 workers in North Carolina, chosen at random by the U.S. Census Bureau:[1]

30 20 10 40 25 20 10 60 15 40 5 30 12 10 10

We aren't surprised that most people estimate their travel time in multiples of five minutes. Here is a stemplot of these data:

0	5
1	0 0 0 0 2 5
2	0 0 5
3	0 0
4	0 0
5	
6	0

The distribution is single-peaked and right-skewed. The longest travel time (60 minutes) may be an outlier. Our goal in this chapter is to describe with numbers the center and variability of this and other distributions.

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2.1 Measuring Center: The Mean

The most common measure of center is the ordinary arithmetic average, or *mean*.

The Mean $\bar{x}$

To find the **mean** of a set of observations, add their values and divide by the number of observations. If the n observations are $x_1, x_2, \ldots, x_n$, their mean is

$$\bar{x} = \frac{x_1 + x_2 + \cdots x_n}{n}$$

or, in more compact notation,

$$\bar{x} = \frac{1}{n}\sum x_i$$

The Σ (capital Greek sigma) in the formula for the mean is short for "add them all up." The subscripts on the observations x_i are just a way of keeping the n observations distinct. They do not necessarily indicate order or any other special facts about the data. The bar over the x indicates the mean of all the x-values. Pronounce the mean $\bar{x}$ as "x-bar." This notation is very common. When writers who are discussing data use $\bar{x}$ or $\bar{y}$, they are talking about a mean.

Statistics In Your World

Don't Hide The Outliers

Data from an airliner's control surfaces, such as the vertical tail rudder, go to cockpit instruments and then to the "black box" flight data recorder. To avoid confusing the pilots, short erratic movements in the data are "smoothed" so that the instruments show overall patterns. When a crash killed 260 people, investigators suspected a catastrophic movement of the tail rudder. But the black box contained only the smoothed data. Sometimes, outliers are more important than the overall pattern.

EXAMPLE 2.1 Travel Times to Work

NCTRAVEL

The mean travel time of our 15 North Carolina workers is

$$\begin{aligned}\bar{x} &= \frac{x_1 + x_2 + \cdots x_{15}}{n}\\ &= \frac{30 + 20 + \cdots + 10}{15}\\ &= \frac{337}{15} = 22.5 \text{ minutes}\end{aligned}$$

In practice, you can enter the data into your calculator and ask for the mean. You don't have to actually add and divide. But you should know that this is what the calculator is doing.

Notice that only 6 of the 15 travel times are larger than the mean. If we leave out the longest single travel time, 60 minutes, the mean for the remaining 14 people is 19.8 minutes. That one observation raises the mean by 2.7 minutes.

Add Bookmark

mean.

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Notice that only 6 of the 15 travel times are larger than the mean. If we leave out the longest single travel time, 60 minutes, the mean for the remaining 14 people is 19.8 minutes. That one observation raises the mean by 2.7 minutes.

Example 2.1 illustrates an important fact about the mean as a measure of center: it is sensitive to the influence of a few extreme observations. These may be outliers, but a skewed distribution that has no outliers will also pull the mean toward its long tail. Because the mean cannot resist the influence of extreme observations, we say that it is not a **resistant measure** of center.

APPLY YOUR KNOWLEDGE

2.1 ***E. Coli* in Swimming Areas.** To investigate water quality, the *Columbus Dispatch* took water specimens at 16 Ohio State Park swimming areas in central Ohio. Those specimens were taken to laboratories and tested for *E. coli*, which are bacteria that can cause serious gastrointestinal problems. For reference, if a 100-milliliter specimen (about 3.3 ounces) of water contains more than 130 *E. coli* bacteria, it is considered unsafe. Here are the *E. coli* levels per 100 milliliters found by the laboratories:[2]

ECOLI

291.0	10.9	47.0	86.0	44.0	18.9	1.0	50.0
190.4	45.7	28.5	18.9	16.0	34.0	8.6	9.6

Find the mean *E. coli* level. How many of the lakes have *E. coli* levels greater than the mean? What feature of the data explains the fact that the mean is greater than most of the observations?

2.2 **Health Care Spending.** Table 1.3 (page 32) gives the 2013 health care expenditure per capita in 35 countries with the highest gross domestic product in 2013. The United States, at 9146 international dollars per person, is a high outlier. Find the mean health care spending in these nations with and without the United States. How much does the one outlier increase the mean? HEALTH

2.2 Measuring Center: The Median

In Chapter 1, we used the midpoint of a distribution as an informal measure of center and gave a method for its computation. The *median* is the formal version of the midpoint, and we now provide a more detailed rule for its calculation.

The Median *M*

The **median** M is the midpoint of a distribution, the number such that half the observations are smaller and the other half are larger. To find the median of a distribution:

1. Arrange all observations in order of size, from smallest to largest.
2. If the number of observations n is odd, the median M is the center observation in the ordered list. If the number of observations n is even, the median M is midway between the two center observations in the ordered list.
3. You can always locate the median in the ordered list of observations by counting up $(n + 1)/2$ observations from the start of the list.

Note that the formula $(n + 1)/2$ does not give the median, just the location of the median in the ordered list. Medians require little arithmetic, so they are easy to find by hand for small sets of data. Arranging even a moderate number of observations in order is very tedious, however, so finding the median by hand for larger sets of data is unpleasant. Even simple calculators have an button, but you will need to use software or a graphing calculator to automate finding the median.

EXAMPLE 2.2 Finding the Median: Odd *n*

What is the median travel time for our 15 North Carolina workers? Here are the data arranged in order:

NCTRAVEL

5 10 10 10 10 12 15 **20** 20 25 30 30 40 40 60

The count of observations, $n = 15$, is odd. The bold **20** is the center observation in the ordered list, with seven observations to its left and seven to its right. This is the median, $M = 20$ minutes.

Because $n = 15$, our rule for the location of the median gives

$$\text{location of } M = \frac{n+1}{2} = \frac{16}{2} = 8$$

That is, the median is the eighth observation in the ordered list. It is faster to use this rule than to locate the center by eye.

EXAMPLE 2.3 Finding the Median: Even *n*

NYTRAVEL

Mitchell Funk/Getty Images

Travel times to work in New York State are (on the average) longer than in North Carolina. Here are the travel times in minutes of 20 randomly chosen New York workers:

10 30 5 25 40 20 10 15 30 20 15 20 85 15 65 15 60 60 40 45

A stemplot not only displays the distribution but also makes finding the median easy because it arranges the observations in order:

0	5
1	0 0 5 5 5 5
2	0 0 **0 5**
3	0 0
4	0 0 5
5	
6	0 0 5
7	
8	5

Moore/Notz/Fligner, *The Basic Practice of Statistics*, 8e, © 2018 W.H. Freeman and Company

The distribution is single-peaked and right-skewed, with several travel times of an hour or more. There is no center observation, but there is a center pair. These are the bold **20** and **25** in the stemplot, which have nine observations before them in the ordered list and nine after them. The median is midway between these two observations:

$$M = \frac{20+25}{2} = 22.5 \text{ minutes}$$

With $n = 20$, the rule for locating the median in the list gives

$$\text{location of } M = \frac{n+1}{2} = \frac{21}{2} = 10.5$$

The location 10.5 means "halfway between the 10th and 11th observations in the ordered list." That agrees with what we found by eye.

2.3 Comparing the Mean and the Median

Examples 2.1 and 2.2 illustrate an important difference between the mean and the median. The median travel time (the midpoint of the distribution) is 20 minutes. The mean travel time is higher, 22.5 minutes. The mean is pulled toward the right tail of this right-skewed distribution. The median, unlike the mean, is *resistant*. If the longest travel time were 600 minutes rather than 60 minutes, the mean would increase to more than 58 minutes, but the median would not change at all. The outlier just counts as one observation above the center, no matter how far above the center it lies. The mean uses the actual value of each observation and so will chase a single large observation upward. The *Mean and Median* applet is an excellent way to compare the resistance of M and $\bar{x}$.

Comparing the Mean and the Median

The mean and median of a roughly symmetric distribution are close together. If the distribution is exactly symmetric, the mean and median are exactly the same. In a skewed distribution, the mean is usually farther out in the long tail than the median.[3]

Many economic variables have distributions that are skewed to the right. For example, the median endowment of colleges and universities in the United States and Canada in 2015 was almost $116 million—but the mean endowment was about $648 million. Most institutions have modest endowments, but a few are very wealthy. Harvard's endowment was more than $36 billion.[4] The few wealthy institutions pull the mean up but do not affect the median. Reports about incomes and other strongly skewed distributions usually give the median ("midpoint") rather than the mean ("arithmetic average"). However, a county that is about to impose a tax of 1% on the incomes of its residents cares about the mean income, not the median. The tax revenue will be 1% of total income, and because the total income is the mean income times the number of residents, the tax revenue can be computed easily from the mean. The mean and median measure center in different ways, and both are useful. *Don't confuse the "average" value of a variable (the mean) with its "typical" value, which we might describe by the median.*

APPLY YOUR KNOWLEDGE

2.3 New York Travel Times. Find the mean of the travel times to work for the 20 New York workers in Example 2.3. Compare the mean and median for these data. What general fact does your comparison illustrate? NYTRAVEL

2.4 New House Prices. The mean and median sales prices of new homes sold in the United States in February 2016 were $301,400 and $348,900, respectively.[5] Which of these numbers is the mean and which is the median? Explain how you know.

2.5 Carbon Dioxide Emissions. Burning fuels in power plants and motor vehicles emits carbon dioxide (CO_2), which contributes to global warming. The CO_2 emissions (metric tons per capita) for countries varies from 0.02 in Burundi to 44.02 in Qatar. Although the data set includes 214 countries, the CO_2 emissions of 15 countries are not available on the World Bank database. The data set is too large to print here, but here are the data for the first five countries:[6] CO2EMISS

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Country	CO_2 Emissions (Metric Tons per Capita)
Aruba	23.92
Andorra	5.97
Afghanistan	0.43
Angola	1.35
Albania	1.61

Find the mean and the median for the full data set. Make a histogram of the data. What features of the distribution explain why the mean is larger than the median?

Albert Valles/Getty Images

2.4 Measuring Variability: The Quartiles

The mean and median provide two different measures of the center of a distribution. But a measure of center alone can be misleading. The U.S. Census Bureau reports that in 2014 the median income of American households was \$53,657. Half of all households had incomes below \$53,657 and half had higher incomes. The mean was much higher, \$75,738, because the distribution of incomes is skewed to the right. But the median and mean don't tell the whole story. The bottom 20% of households had incomes less than \$21,432, and households in the top 5% took in more than \$206,568.[7] We are interested in the *variability* of incomes as well as their center. *The simplest useful numerical description of a distribution requires both a measure of center and a measure of variability.*

One way to measure variability is to give the smallest and largest observations. For example, the travel times of our 15 North Carolina workers range from 5 minutes to 60 minutes. These single observations show the full variability of the data, but they may be outliers. We can improve our description of variability by also looking at the variability of the middle half of the data. The *quartiles* mark out the middle half. Count up the ordered list of observations, starting from the smallest. The *first quartile* lies one-quarter of the way up the list. The *third quartile* lies three-quarters of the way up the list. In other words, the first quartile is larger than 25% of the observations, and the third quartile is larger than 75% of the observations. The second quartile is the median, which is larger than 50% of the observations. That is the idea of quartiles. We need a rule to make the idea exact. The rule for calculating the quartiles uses the rule for the median.

The Quartiles Q_1 and Q_3

To calculate the **quartiles:**

1. Arrange the observations in increasing order and locate the median, *M*, in the ordered list of observations.
2. The **first quartile**, $\mathbf{Q_1}$, is the median of the observations whose position in the ordered list is to the left of the location of the overall median.
3. The **third quartile**, $\mathbf{Q_3}$, is the median of the observations whose position in the ordered list is to the right of the location of the overall median.

Here are examples that show how the rules for the quartiles work for both odd and even numbers of observations.

EXAMPLE 2.4 Finding the Quartiles: Odd *n*

NCTRAVEL

Our North Carolina sample of 15 workers' travel times, arranged in increasing order, is

5 10 10 10 10 12 15 **20** 20 25 30 30 40 40 60

There is an odd number of observations, so the median is the middle one, the bold **20** in the list. The first quartile is the median of the seven observations to the left of the median. This is the fourth of these seven observations, so Q_1 = 10 minutes. If you want, you can use the rule for the location of the median with $n = 7$:

$$\text{location of } Q_1 = \frac{n+1}{2} = \frac{7+1}{2} = 4$$

The third quartile is the median of the seven observations to the right of the median, Q_3 = 30 minutes. *When there is an odd number of observations, leave out the overall median when you locate the quartiles in the ordered list.*

The quartiles are *resistant* because they are not affected by a few extreme observations. For example, Q_3 would still be 30 if the outlier were 600 rather than 60.

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The quartiles are *resistant* because they are not affected by a few extreme observations. For example, Q_3 would still be 30 if the outlier were 600 rather than 60.

EXAMPLE 2.5 Finding the Quartiles: Even *n*

NCTRAVEL

Here are the travel times to work of the 20 New Yorker workers from Example 2.3, arranged in increasing order:

5 10 10 15 15 15 15 20 | 20 20 25 30 30 40 40 45 60 60 65 85

There is an even number of observations, so the median lies midway between the middle pair, the 10th and 11th in the list. Its value is M = 22.5 minutes. We have marked the location of the median by |. The first quartile is the median of the first 10 observations because these are the observations to the left of the location of the median. Check that Q_1 = 15 minutes and Q_3 = 42.5 minutes. *When the number of observations is even, include all the observations when you locate the quartiles.*

Be careful when, as in these examples, several observations take the same numerical value. Write down all the observations, arrange them in order, and apply the rules just as if they all had distinct values.

There are several rules for finding the quartiles. Some calculators and software use rules that give results that differ from ours for some sets of data (see Example 2.8). Our rule is the simplest for hand calculation, with the results from the various rules generally being close to each other.

2.5 The Five-Number Summary and Boxplots

The smallest and largest observations tell us little about the distribution as a whole, but they give information about the tails of the distribution that is missing if we know only the median and the quartiles. To get a quick summary of both center and variability, combine all five numbers.

The Five-Number Summary

The **five-number summary** of a distribution consists of the smallest observation, the first quartile, the median, the third quartile, and the largest observation, written in order from smallest to largest. In symbols, the five-number summary is

$$\text{Minimum} \quad Q_1 \quad M \quad Q_3 \quad \text{Maximum}$$

These five numbers offer a reasonably complete description of center and variability. The five-number summaries of travel times to work from Examples 2.4 and 2.5 are

North Carolina	5	10	20	30	60
New York	5	15	22.5	42.5	85

The five-number summary of a distribution leads to a new graph, the *boxplot*. Figure 2.1 shows boxplots comparing travel times to work in North Carolina and New York.

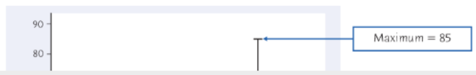
Boxplot

A **boxplot** is a graph of the five-number summary.

- A central box spans the quartiles Q_1 and Q_3.
- A line in the box marks the median M.
- Lines extend from the box out to the smallest and largest observations.

Because boxplots show less detail than histograms or stemplots, they are best used for side-by-side comparison of more than one distribution, as in Figure 2.1. Be sure to include a numerical scale in the graph. When you look at a boxplot, first locate the median, which marks the center of the distribution. Then look at the variability. The span of the central box shows the variability of the middle half of the data, and the extremes (the smallest and largest observations) show the variability of the entire data set. We see from Figure 2.1 that travel times to work are in general a bit longer in New York than in North Carolina. The median, both quartiles, and the maximum are all larger in New York. New York travel times are also more variable, as shown by the span of the box and the difference between the extremes. Note that the boxes with arrows in Figure 2.1 that indicate the location of the five-number summary are *not* part of the boxplot but are included purely for illustration.

Finally, the New York data are more strongly right-skewed. In a symmetric distribution, the first and third quartiles are equally distant from the median. In most distributions that are skewed to the right, on the other hand, the third quartile will be farther above the median than the first quartile is below it. The extremes behave the same way, but remember that they are just single observations and may say little about the distribution as a whole.



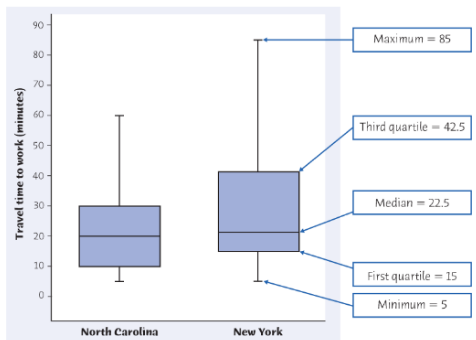


Figure 2.1
Moore/Notz/Fligner, *The Basic Practice of Statistics*, 8e, © 2018 W.H. Freeman and Company

FIGURE 2.1
Boxplots comparing the travel times to work of samples of workers in North Carolina and New York.

Macmillan Learning Online Resources

- The StatClips Examples video, *Exploratory Pictures for Quantitative Data Example C*, provides a worked-out example of computing the five-number summary for both odd and even sample sizes, and then draws a comparative boxplot.

APPLY YOUR KNOWLEDGE

2.6 Shared Pain and Bonding. Although painful experiences are involved in social rituals in many parts of the world, little is known about the social effects of pain. Will sharing painful experiences in a small group lead to greater bonding of group members than sharing a similar non-painful experience? Fifty-four university students in South Wales were divided at random into a pain group containing 27 students, with the remaining students in the no-pain group. Pain was induced by two tasks. In the first task, students submerged their hands in freezing water for as long as possible, moving metal balls at the bottom of the vessel into a submerged container; in the second task, students performed a standing wall squat with back straight and knees at 90 degrees for as long as possible. The no-pain group completed the first task using room temperature water for 90 seconds and the second task by balancing on one foot for 60 seconds, changing feet if necessary. In both the pain and no-pain settings, the students completed the tasks in small groups, which typically consisted of four students and contained similar levels of group interaction. Afterward, each student completed a questionnaire to create a bonding score based on answers to questions such as "I feel the participants in this study have a lot in common," or "I feel I can trust the other participants." Here are the bonding scores for the two groups:[8] DATA BONDING

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No-pain group:	3.43	4.86	1.71	1.71	3.86	3.14	4.14	3.14	4.43	3.71
	3.00	3.14	4.14	4.29	2.43	2.71	4.43	3.43	1.29	1.29
	3.00	3.00	2.86	2.14	4.71	1.00	3.71			
Pain group:	4.71	4.86	4.14	1.29	2.29	4.43	3.57	4.43	3.57	3.43
	4.14	3.86	4.57	4.57	4.29	1.43	4.29	3.57	3.57	3.43
	2.29	4.00	4.43	4.71	4.71	2.14	3.57			

(a) Find the five-number summaries for the pain and the no-pain groups.

(b) Construct a comparative boxplot for the two groups following the model of Figure 2.1. It doesn't matter if your boxplots are horizontal or vertical, but they should be drawn on the same set of axes.

(c) Which group tends to have higher bonding scores? Is the variability in the two groups similar, or does one of the groups tend to have less variable bonding scores? Does either group contain one or more clear outliers?

2.7 **Fuel Economy for Midsize Cars.** The Department of Energy provides fuel economy ratings for all cars and light trucks sold in the United States. Here are the estimated miles per gallon for city driving for the 186 cars classified as midsize in 2016, arranged in increasing order:[9] MIDCARS

11	11	11	12	13	13	13	14	14	14	14	14	15	15	15	15	15	15
16	16	16	16	16	16	16	16	16	16	16	16	16	17	17	17	17	17
17	18	18	18	18	18	18	18	18	18	19	19	19	19	19	19	19	19
20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
21	21	21	21	21	21	21	21	21	22	22	22	22	22	22	22	22	22
22	22	22	22	22	22	22	23	23	23	23	23	23	23	24	24	24	24
24	24	24	24	24	25	25	25	25	25	25	25	25	25	25	25	25	25
25	25	26	26	26	26	26	26	26	26	26	26	26	27	27	27	27	27
27	27	27	27	27	27	27	27	28	28	28	28	28	28	28	28	28	28
29	29	29	29	29	29	30	30	30	30	31	31	35	36	39	40	40	40
40	41	43	44	54	58												

(a) Give the five-number summary of this distribution.

(b) Draw a boxplot of these data. What is the shape of the distribution shown by the boxplot? Which features of the boxplot led you to this conclusion? Are any observations unusually small or large?

2.6 Spotting Suspected Outliers and Modified Boxplots*

Look again at the stemplot of travel times to work in New York in Example 2.3. The five-number summary for this distribution is

5 15 22.5 42.5 85

How shall we describe the variability of this distribution? The smallest and largest observations are extremes that don't describe the variability of the majority of the data. The distance between the quartiles (the range of the center half of the data) is a more resistant measure of variability. This distance is called the *interquartile range*.

The Interquartile Range (*IQR*)

The **interquartile range** (**IQR**) is the distance between the first and third quartiles,

$$IQR = Q_3 - Q_1$$

How Much Is That House Worth?

The town of Manhattan, Kansas, is sometimes called "the Little Apple" to distinguish it from that other Manhattan, "the Big Apple." A few years ago, a house there appeared in the county appraiser's records valued at $200,059,000. That would be quite a house even on Manhattan Island. As you might guess, the entry was wrong: the true value was $59,500. But before the error was discovered, the county, the city, and the school board had based their budgets on the total appraised value of real estate, which the one outlier jacked up by 6.5%. It can pay to spot outliers before you trust your data.

For our data on New York travel times, $IQR = 42.5 - 15 = 27.5$ minutes. However, *no single numerical measure of variability, such as IQR, is very useful for describing skewed distributions*. The two sides of a skewed distribution have different variability, so one number can't summarize them. That's why we give the full five-number summary. The interquartile range is mainly used as the basis for a rule of thumb for identifying suspected outliers.

The 1.5 × *IQR* Rule for Outliers

Call an observation a suspected outlier if it falls more than $1.5 \times IQR$ above the third quartile or below the first quartile.

EXAMPLE 2.6 Using the 1.5 × *IQR* Rule

NYTRAVEL

For the New York travel time data, $IQR = 27.5$ and

$$1.5 \times IQR = 1.5 \times 27.5 = 41.25$$

Any values not falling between

$$Q_1 - (1.5 \times IQR) = 15.0 - 41.25 = -26.25 \quad \text{and}$$
$$Q_3 + (1.5 \times IQR) = 42.5 + 41.25 = 83.75$$

are flagged as suspected outliers. Look again at the stemplot in Example 2.3 (page 50): the only suspected outlier is the longest travel time, 85 minutes. The 1.5 × *IQR* rule suggests that the three next-longest travel times (60, 60, and 65 minutes) are just part of the long right tail of this skewed distribution.

Add Bookmark

$$Q_3 + (1.5 \times IQR) = 42.5 + 41.25 = 83.75$$

are flagged as suspected outliers. Look again at the stemplot in Example 2.3 (page 50): the only suspected outlier is the longest travel time, 85 minutes. The 1.5 × *IQR* rule suggests that the three next-longest travel times (60, 60, and 65 minutes) are just part of the long right tail of this skewed distribution.

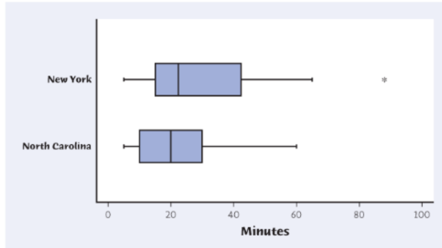

Figure 2.2

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FIGURE 2.2

Horizontal modified boxplots comparing the travel times to work of samples of workers in North Carolina and New York.

In a modified boxplot, which is provided by many software packages, the suspected outliers are identified in the boxplot with a special plotting symbol such as an asterisk (*). Comparing Figure 2.2 with Figure 2.1, we see that the largest observation from New York is flagged as an outlier. The line beginning at the third quartile no longer extends to the maximum, but now ends at 65, which is the largest observation from New York that is not identified as an outlier. Figure 2.2 also displays the modified boxplots horizontally rather than vertically, an option available in some software packages that does not change the interpretation of the plot. Finally, the 1.5 × *IQR* rule is not a replacement for looking at the data. It is most useful when large volumes of data are processed automatically.

APPLY YOUR KNOWLEDGE

2.8 Travel Time to Work. In Example 2.1 (page 48), we noted the influence of one long travel time of 60 minutes in our sample of 15 North Carolina workers. Does the 1.5 × *IQR* rule identify this travel time as a suspected outlier? NCTRAVEL

2.9 Fuel Economy for Midsize Cars. Exercise 2.7 (page 55) gives the estimated miles per gallon (mpg) for city driving for the 186 cars classified as midsize in 2016. In that exercise, we noted that several of the mpg values were unusually large. Which of these are suspected outliers by the 1.5 × *IQR* rule? Although outliers can be produced by errors or incorrectly recorded observations, they are often observations that differ from the others in some particular way. In this case, the cars producing the high outliers share a common feature. What do you think that is? MIDCARS

2.7 Measuring Variability: The Standard Deviation

The five-number summary is not the most common numerical description of a distribution. That distinction belongs to the combination of the mean to measure center and the *standard deviation* to measure variability. The standard deviation and its close relative, the *variance,* measure variability by looking at how far the observations are from their mean.

The Standard Deviation s

The **variance** s^2 of a set of observations is an average of the squares of the deviations of the observations from their mean. In symbols, the variance of n observations $x_1, x_2, \ldots, x_n$ is

$$s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n - 1}$$

or, more compactly,

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

The **standard deviation** s is the square root of the variance s^2:

$$s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

In practice, use software or your calculator to obtain the standard deviation from keyed-in data. Doing an example step-by-step will help you understand how the variance and standard deviation work, however.

EXAMPLE 2.7 Calculating the Standard Deviation

SATMATH

Georgia Southern University had 2786 students with regular admission in its freshman class of 2015. For each student, data are available on their SAT and ACT scores (if taken), high school GPA, and the college within the university to which they were admitted.[10] In Exercise 3.49 (page 98), the full data set for the SAT Mathematics scores will be examined. Here are the first five observations from that data set:

490 580 450 570 650

We will compute $\bar{x}$ and s for these students. First find the mean:

$$\bar{x} = \frac{490 + 580 + 450 + 570 + 650}{5} = \frac{2740}{5} = 548$$

Figure 2.3 displays the data as points above the number line, with their mean marked by an asterisk (*). The arrows mark two of the deviations from the mean. The deviations show how variable the data are about their mean. They are the starting point for calculating the variance and the standard deviation.

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Observations x_i	Deviations $x_i - \bar{x}$	Squared Deviations $(x_i - \bar{x})^2$
490	490 − 548 = −58	$(-58)^2$ = 3,364
580	580 − 548 = 32	32^2 = 1,024
450	450 − 548 = −98	$(-98)^2$ = 9,604
570	570 − 548 = 22	22^2 = 484
650	650 − 548 = 102	102^2 = 10,404
	sum = 0	sum = 24,880

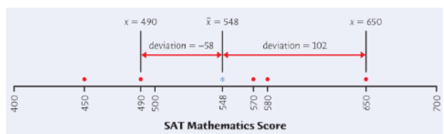

Figure 2.3
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FIGURE 2.3

SAT Mathematics scores for five students, with their mean (*) and the deviations of two observations from the mean shown, for Example 2.7.

The variance is the sum of the squared deviations divided by one less than the number of observations:

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = \frac{24,880}{4} = 6220$$

The standard deviation is the square root of the variance:

$$s = \sqrt{6220} = 78.87$$

Notice that the "average" in the variance s^2 divides the sum by one fewer than the number of observations, that is, $n - 1$ rather than n. The reason is that the deviations $x_i - \bar{x}$ always sum to exactly 0 so that knowing $n - 1$ of them determines the last one. Only $n - 1$ of the squared deviations can vary freely, and we average by dividing the total by $n - 1$. The number $n - 1$ is called the **degrees of freedom** of the variance or standard deviation. Some calculators offer a choice between dividing by n and dividing by $n - 1$, so be sure to use $n - 1$.

More important than the details of hand calculation are the properties that determine the usefulness of the standard deviation:

- s measures *variability about the mean* and should be used only when the mean is chosen as the measure of center.
- s is *always zero or greater than zero.* $s = 0$ only when there is no variability. This happens only when all observations have the same value. Otherwise, $s > 0$. As the observations become more variable about their mean, s gets larger.
- s has the *same units of measurement as the original observations.* For example, if you measure weight in kilograms, both the mean $\bar{x}$ and the standard deviation s are also in kilograms. This is one reason to prefer s to the

650	650 − 548 = 102	102^2 = 10,404
	sum = 0	sum = 24,880

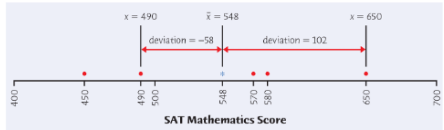

Figure 2.3
Moore/Notz/Fligner, *The Basic Practice of Statistics*, 8e, © 2018 W.H. Freeman and Company

FIGURE 2.3
SAT Mathematics scores for five students, with their mean (*) and the deviations of two observations from the mean shown, for Example 2.7.

The variance is the sum of the squared deviations divided by one less than the number of observations:

$$s^2 = \frac{1}{n-1}\sum(x_i - x)^2 = \frac{24,880}{4} = 6220$$

The standard deviation is the square root of the variance:

$$s = \sqrt{6220} = 78.87$$

Notice that the "average" in the variance s^2 divides the sum by one fewer than the number of observations, that is, $n-1$ rather than n. The reason is that the deviations $x_i - \bar{x}$ always sum to exactly 0 so that knowing $n-1$ of them determines the last one. Only $n-1$ of the squared deviations can vary freely, and we average by dividing the total by $n-1$. The number $n-1$ is called the **degrees of freedom** of the variance or standard deviation. Some calculators offer a choice between dividing by n and dividing by $n-1$, so be sure to use $n-1$.

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- s has the *same units of measurement as the original observations.* For example, if you measure weight in kilograms, both the mean $\bar{x}$ and the standard deviation s are also in kilograms. This is one reason to prefer s to the variance s^2, which would be in squared kilograms.
- Like the mean $\bar{x}$, s is *not resistant.* A few outliers can make s very large.

The use of squared deviations renders s even more sensitive than $\bar{x}$ to a few extreme observations. For example, the standard deviation of the travel times for the 15 North Carolina workers in Example 2.1 is 15.23 minutes. (Use your calculator or software to verify this.) If we omit the high outlier, the standard deviation drops to 11.56 minutes.

If you feel that the importance of the standard deviation is not yet clear, you are right. We will see in Chapter 3 that the standard deviation is the natural measure of variability for a very important class of symmetric distributions, the Normal distributions. The usefulness of many statistical procedures is tied to distributions of particular shapes. This is certainly true of the standard deviation.

2.8 Choosing Measures of Center and Variability

We now have a choice between two descriptions of the center and variability of a distribution: the five-number summary $\bar{x}$ or and s. Because $\bar{x}$ and s are sensitive to extreme observations, they can be misleading when a distribution is strongly skewed or has outliers. In fact, because the two sides of a skewed distribution have different variability, no single number describes the variability well. The five-number summary, with its two quartiles and two extremes, does a better job.

Choosing a Summary

- The five-number summary is usually better than the mean and standard deviation for describing a skewed distribution or a distribution with strong outliers.
- Use $\bar{x}$ and s only for reasonably symmetric distributions that are free of outliers.

Outliers can greatly affect the values of the mean $\bar{x}$ and the standard deviation s, the most common measures of center and variability. Many more elaborate statistical procedures also can't be trusted when outliers are present. *Whenever you find outliers in your data, try to find an explanation for them.* Sometimes the explanation is as simple as a typing error, such as typing 10.1 as 101; if this is the case, correct the typing error. Sometimes a measuring device broke down or a subject gave a frivolous response, like the student in a class survey who claimed to study 30,000 minutes per night. (Yes, that really happened.) In all these cases, you can simply remove the outlier from your data. When outliers are "real data," like the long travel times of some New York workers, you should choose statistical methods that are not greatly disturbed by the outliers. For example, use the five-number summary rather than and s to describe a distribution with extreme outliers. We will meet other examples later in the book.

Remember that a graph gives the best overall picture of a distribution. If data have been entered into a calculator or statistical program, it is very simple and quick to create several graphs to see all the different features of a distribution. Numerical measures of center and variability report specific facts about a distribution, but they do not describe its entire shape. Numerical summaries do not disclose the presence of multiple peaks or clusters, for example. Exercise 2.11 shows how misleading numerical summaries can be. **Always plot your data**.

Macmillan Learning Online Resources

- The Snapshots video, *Summarizing Quantitative Data*, provides an overview of the need for measures of center and variability as well as some details of the computations.
- The StatClips Examples video, *Summaries of Quantitative Data Example C*, gives the details for the computation of the mean, median, and standard deviation in a small example. You can verify the computations along with the video, either by hand or using your technology.
- The StatClips Examples videos, *Basic Principles of Exploring Data Example B* and *Basic Principles of Exploring Data Example C*, emphasize the need to examine outliers and understand them, rather than simply discarding observations that don't seem to fit.

APPLY YOUR KNOWLEDGE

2.10 $\bar{x}$ and s by Hand. Radon is a naturally occurring gas and is the second leading cause of lung cancer in the United States.[11] It comes from the natural breakdown of uranium in the soil and enters buildings through cracks and other holes in the foundations. Found throughout the United States, levels vary considerably from state to state. Several methods can reduce the levels of radon in your home, and the Environmental Protection Agency recommends using one of these if the measured level in your home is above 4 picocuries per liter. Four readings from Franklin County, Ohio, where the county average is 8.4 picocuries per liter, were 7.1, 11.6, 8.1, and 13.4.

(a) Find the mean step-by-step. That is, find the sum of the four observations and divide by 4.

(b) Find the standard deviation step-by-step. That is, find the deviation of each observation from the mean, square the deviations, then obtain the variance and the standard deviation. Example 2.7 (page 58) shows the method.

(c) Now enter the data into your calculator and use the mean and standard deviation buttons to obtain $\bar{x}$ and s. Do the results agree with your hand calculations?

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(c) Now enter the data into your calculator and use the mean and standard deviation buttons to obtain $\bar{x}$ and s. Do the results agree with your hand calculations?

Doug Martin/Getty Images

2.11 $\bar{x}$ and s Are Not Enough. The mean and standard deviation s measure center and variability but are not a complete description of a distribution. Data sets with different shapes can have the same mean and standard deviation. To demonstrate this fact, use your calculator to find $\bar{x}$ and s for these two small data sets. Then make a stemplot of each and comment on the shape of each distribution. DATA SET2

Data A:	9.14	8.14	8.74	8.77	9.26	8.10	6.13	3.10	9.13	7.26	4.74
Data B:	6.58	5.76	7.71	8.84	8.47	7.04	5.25	5.56	7.91	6.89	12.50

2.12 Choose a Summary. The shape of a distribution is a rough guide to whether the mean and standard deviation are a helpful summary of center and variability. For which of the following distributions would $\bar{x}$ and s be useful? In each case, give a reason for your decision.

(a) Percents of high school graduates in the states taking the SAT, Figure 1.8 (page 27)

(b) Iowa Tests scores, Figure 1.7 (page 27)

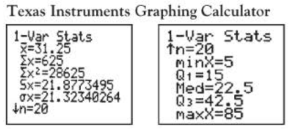
(c) New York travel times, Figure 2.1 (page 54)

2.9 Examples of Technology

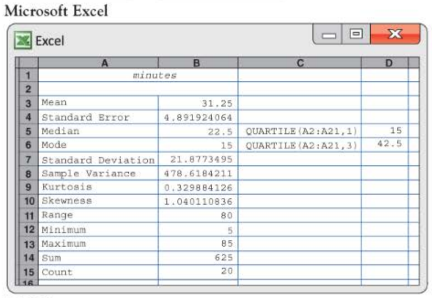
Although a calculator with "two-variable statistics" functions will do many of the basic calculations we need throughout the text, more elaborate technology is helpful. Graphing calculators and computer software will do calculations and make graphs as you command, freeing you to concentrate on choosing the right methods and interpreting your results. Figure 2.4 displays outputs from three technology tools for describing the travel times to work of 20 people in New York State (Example 2.3, page 50). Can you find $\bar{x}$ s, and the five-number summary in each output? The big message is: *once you know what to look for, you can read output from any technological tool.*

The displays in Figure 2.4 come from a Texas Instruments graphing calculator, JMP statistical software, and the Microsoft Excel spreadsheet program. JMP allows you to choose what descriptive measures you want, whereas the descriptive measures in Excel and the calculator give some things we don't need. Just ignore the extras. Because Excel's "Descriptive Statistics" menu item doesn't give the quartiles, we used the spreadsheet's separate quartile function to get Q_1 and Q_3.

Texas Instruments Graphing Calculator



Microsoft Excel



	A	B	C	D
1	minutes			
2				
3	Mean	31.25		
4	Standard Error	4.891924064		
5	Median	22.5	QUARTILE(A2:A21,1)	15
6	Mode	15	QUARTILE(A2:A21,3)	42.5
7	Standard Deviation	21.8773495		
8	Sample Variance	478.6184211		
9	Kurtosis	0.329884126		
10	Skewness	1.040110836		
11	Range	80		
12	Minimum	5		
13	Maximum	85		
14	Sum	625		
15	Count	20		

JMP Output

12	Minimum	5			
13	Maximum	85			
14	Sum	625			
15	Count	20			

JMP Output

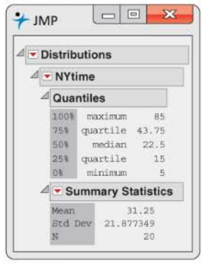


Figure 2.4

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FIGURE 2.4

Output from a graphing calculator, a statistical software package, and a spreadsheet program describing the data on travel times to work in New York State.

EXAMPLE 2.8 What Is the Third Quartile?

In Example 2.5, we saw that the quartiles of the New York travel times are $Q_1 = 15$ and $Q_3 = 42.5$. Look at the output displays in Figure 2.4. The calculator and Excel agree with our work, while JMP says that $Q_3 = 43.75$. What happened? *There are several rules for finding the quartiles. Some calculators and software use rules that give results different from ours for some sets of data.* This is true of JMP and Excel, although Excel agrees with our work in this example. Results from the various rules are generally close to each other, so the differences are not important in practice. Our rule is the simplest for hand calculation.

Macmillan Learning Online Resources

There are online resources to help you use technology.

- The statistical software packages CrunchIt! and JMP are available online.
- The Video Technology Manuals for Minitab, the TI graphing calculator, Excel, JMP, CrunchIt!, R, and SPSS provide explicit instructions for producing output similar to that provided in Figure 2.4 for the particular technology you are using.

2.10 Organizing a Statistical Problem

Most of our examples and exercises have aimed to help you learn basic tools (graphs and calculations) for describing and comparing distributions. You have also learned principles that guide use of these tools, such as "start with a graph" and "look for the overall pattern and striking deviations from the pattern." The data you work with are not just numbers—they describe specific settings such as water depth in the Everglades or travel time to work. Because data come from a specific setting, the final step in examining data is a *conclusion for that setting*. Water depth in the Everglades has a yearly cycle that reflects Florida's wet and dry seasons. Travel times to work are generally longer in New York than in North Carolina.

Let's return to the on-time high school graduation rates discussed in Example 1.4 (page 21). We know from the example that the on-time graduation rates vary from 61.4% in the District of Columbia to 90.5% in Iowa, with a median of 83.9%. State graduation rates are related to many factors, and in a statistical problem, we often try to explain the differences or variation in a variable such as graduation rate by some of these factors. For example, do states with lower household incomes tend to have lower high school graduation rates? Or, do the states in some regions of the country tend to have lower high school graduation rates than in other regions?

As you learn more statistical tools and principles, you will face more complex statistical problems. Although no framework accommodates all the varied issues that arise in applying statistics to real settings, we find the following four-step thought process gives useful guidance. In particular, the first and last steps emphasize that statistical problems are tied to specific real-world settings and therefore involve more than doing calculations and making graphs.

Organizing a Statistical Problem: A Four-Step Process

STATE: What is the practical question, in the context of the real-world setting?
PLAN: What specific statistical operations does this problem call for?
SOLVE: Make the graphs and carry out the calculations needed for this problem.
CONCLUDE: Give your practical conclusion in the setting of the real-world problem.

To help you master the basics, many exercises will continue to tell you what to do—make a histogram, find the five-number summary, and so on. Real statistical problems don't come with detailed instructions. From now on, especially in the later chapters of the book, you will meet some exercises that are more realistic. Use the four-step process as a guide to solving and reporting these problems. They are marked with the four-step icon, as the following example illustrates.

EXAMPLE 2.9 Comparing Graduation Rates

GRADRATE

STATE: Federal law requires all states in the United States to use a common computation of on-time high school graduation rates beginning with the 2010–2011 school year. Previously, states chose one of several computation methods that gave answers that could differ by more than 10%. This common computation allows for meaningful comparison of graduation rates between the states.

We know from Table 1.1 (page 22), that the on-time high school graduation rates in the 2013–2014 school year varied from 61.4% in the District of Columbia to 90.5% in Iowa. The U.S. Census Bureau divides the 50 states and the District of Columbia into four geographical regions: the Northeast (NE), Midwest (MW), South (S), and West (W). The region for each state is included in Table 1.1. Do the states in the four regions of the country display distinct distributions of graduation rates? How do the mean graduation rates of the states in each of these regions compare?

PLAN: Use graphs and numerical descriptions to describe and compare the distributions of on-time high school graduation rates of the states in the four regions of the United States.

SOLVE: We might use boxplots to compare the distributions, but stemplots preserve more detail and work well for data sets of these sizes. Figure 2.5 displays the stemplots with the stems lined up for easy comparison. The stems have been split to better display the distributions, and the data have been rounded to the nearest percent. The stemplots overlap, and some care is needed when comparing the four stemplots because the sample sizes differ, with some stemplots having more leaves than others. The states in the Northeast and Midwest have distributions that are similar to each other. The South, with the most observations, has one low observation corresponding to the District of Columbia that stands apart from the others and some skewness to the left. Because the District of Columbia is not a state, although often included with state data, we have reported summary statistics for the South with and without this observation. Excluding the District of Columbia, the South is similar to the Northeast and Midwest, although with a distribution that tends to have slightly lower graduation rates and a little more variability. The West tends to have lower graduation rates than the other three regions, with greater variability. With little skewness and no serious outliers other than the District of Columbia, we report $\bar{x}$ and s as our summary measures of center and variability of the distribution of the on-time graduation rates of the states in each region:

Region	N	Mean	Standard Deviation
Midwest	12	85.60	3.72
Northeast	9	85.36	3.65
South (including DC)	17	81.66	7.20
South (excluding DC)	16	82.93	5.13
West	13	76.98	5.36

Midwest		Northeast		South		West	
6		6		6	1	6	
6		6		6		6	9
7		7		7	3	7	012
7	9	7	8	7	568	7	67789
8	123	8	1	8	034	8	124
8	667789	8	6677889	8	556677788	8	5
9	01	9		9		9	

Figure 2.5
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FIGURE 2.5
Stemplots comparing the distributions of graduation rates for the four census regions from Table 1.1, for Example 2.9.

CONCLUDE: The table of summary statistics confirms what we see in the stemplots. The states in the Midwest and Northeast are most similar to each other, with the South, excluding the District of Columbia, having a slightly lower mean and higher standard deviation. The states in the West have a lower mean graduation rate than the other three regions, with a standard deviation similar to the South but higher than the Midwest or Northeast.

It is important to remember that the individuals in Example 2.9 are the states. For example, the mean of 85.36 is the mean of the on-time graduation rates for the nine Northeastern states, and the standard deviation tells us how much these state rates vary about this mean. However, the mean of these nine states *is not* the same as the graduation rate for all high school students in the Northeast, unless the states have the same number of high school students. The graduation rate for all high school students in the Northeast would be a *weighted* average of the state rates, with the larger states receiving more weight. For example, because New York is the most populous state in the Northeast and also has the lowest graduation rate, we would expect the graduation rate of all high school students in the Northeast to be lower than 85.36, because New York would pull down the overall graduation rate. See Exercise 2.37 (page 70) for a similar example.

APPLY YOUR KNOWLEDGE

2.13 **Logging in the Rain Forest.** "Conservationists have despaired over destruction of tropical rain forest by logging, clearing, and burning." These words begin a report on a statistical study of the effects of logging in Borneo.[12] Charles Cannon of Duke University and his coworkers compared forest plots that had never been logged (Group 1) with similar plots nearby that had been logged one year earlier (Group 2) and eight years earlier (Group 3). All plots were 0.1 hectare in area. Here are the counts of trees for plots in each group: LOGGING

Group 1:	27	22	29	21	19	33	16	20	24	27	28	19
Group 2:	12	12	15	9	20	18	17	14	14	2	17	19
Group 3:	18	4	22	15	18	19	22	12	12			

AustralianCamera/Shutterstock

To what extent has logging affected the count of trees? Follow the four-step process in reporting your work.

2.14 **Worldwide Child Mortality.** Although child mortality rates worldwide have dropped by more than 50% since 1990, it is still the case that 16,000 children under five years old die each day. The mortality rates for children under five vary from 1.9 per 1000 in Luxembourg to 156.9 per 1000 in Angola. In Exercise 1.36 (page 41), you were asked to draw a histogram of this data. In this exercise, you will explore the relationship between child mortality and a measure of a country's economic wealth. One measure used by the World Bank is the gross national income (GNI) per capita, the dollar value of a country's final income in a year divided by its population. It reflects the average income of a country's citizens, and the World Bank uses GNI per capita to classify countries into low-income, lower-middle income, upper-middle income, and high-income economies. Although the data set includes 213 countries, the child mortality rates of 22 countries are not available on the World Health Organization database.

Because the data set is too large to print here, we give the data for the first five countries:[13]

MORTALTY

Country	Child Mortality Rate (per 1000)	Economy Classification
Aruba	—	High
Andorra	2.800	High
Afghanistan	91.100	Low
Angola	156.900	Upper-middle
Albania	14.000	Upper-middle

Give a full description of the distribution of child mortality rates for the countries in each of the four economic classifications, and identify any high outliers. Compare the four groups. Does the economic classfication used by the World Bank do a good job of explaining the differences in child mortality rates among the countries?

CHAPTER 2 SUMMARY

Chapter Specifics

- A numerical summary of a distribution should report at least its **center** and its **variability**.
- The **mean** and the **median M** describe the center of a distribution in different ways. The mean is the arithmetic average of the observations, and the median is the midpoint of the values.
- When you use the median to indicate the center of the distribution, describe its variability by giving the **quartiles**. The **first quartile**, Q_1, has one-fourth of the observations below it, and the **third quartile**, Q_3, has three-fourths of the observations below it.
- The **five–number summary** consisting of the median, the quartiles, and the smallest and largest individual observations provides a quick overall description of a distribution. The median describes the center, and the quartiles and extremes show the variability.
- **Boxplots** based on the five-number summary are useful for comparing several distributions. The box spans the quartiles and shows the variability of the central half of the distribution. The median is marked within the box. Lines extend from the box to the extremes and show the full variability of the data.
- The **variance** s^2 and especially its square root, the **standard deviation** s, are common measures of variability about the mean as center. The standard deviation s is zero when there is no variability and gets larger as the variability increases.
- A **resistant measure** of any aspect of a distribution is relatively unaffected by changes in the numerical value of a small proportion of the total number of observations, no matter how large these changes are. The median and quartiles are resistant, but the mean and the standard deviation are not.
- The mean and standard deviation are good descriptions for symmetric distributions without outliers. They are most useful for the Normal distributions introduced in the next chapter. The five-number summary is a better description for skewed distributions.
- Numerical summaries do not fully describe the shape of a distribution. Always plot your data.
- A statistical problem has a real-world setting. You can organize many problems using the following four steps: **state, plan, solve**, and **conclude**.

Link It

In this chapter, we have continued our study of exploratory data analysis. Graphs are an important visual tool for organizing and identifying patterns in data. They give a fairly complete description of a distribution, although for many problems, the important information in your data can be described by a few numbers. These numerical summaries can be useful for describing a single distribution as well as for comparing the distributions from several groups of observations.

Two important features of a distribution are the center and the variability. For distributions that are approximately symmetric without outliers, the mean and standard deviation are important numeric summaries for describing and comparing distributions. But if the distribution is not symmetric and/or has outliers, the five-number summary often provides a better description.

The boxplot gives a picture of the five-number summary that is useful for a simple comparison of several distributions. Remember that the boxplot is based only on the five-number summary and does not have any information beyond these five numbers. Certain features of a distribution that are revealed in histograms and stemplots will not be evident from a boxplot alone. These include gaps in the data and the presence of several peaks. You must be careful when reducing a distribution to a few numbers to make sure that important information has not been lost in the process.

Macmillan Learning Online Resources

If you are having difficulty with any of the sections of this chapter, these online resources should help prepare you to solve the exercises at the end of this chapter:

- StatTutor starts with a video review of each section and asks a series of questions to check your understanding.
- LearningCurve provides you with a series of questions about the chapter geared to your level of understanding.

2.15 The 2013–2014 roster of the Seattle Seahawks, winners of the 2014 NFL Super Bowl, included 10 defensive linemen and nine offensive linemen. The weights in pounds of the 10 defensive linemen were LINEMEN

311 254 297 260 323 242 300 252 303 274

The mean of these data is
(a) 281.60.
(b) 282.50.
(c) 285.50.

2.16 The median of the data in Exercise 2.15 is LINEMEN
(a) 281.60.
(b) 282.50.
(c) 285.50.

2.17 The first quartile of the data in Exercise 2.15 is LINEMEN
(a) 253.
(b) 254.
(c) 297.

2.18 If a distribution is skewed to the left,
(a) the mean is less than the median.
(b) the mean and median are equal.
(c) the mean is greater than the median.

2.19 What percent of the observations in a distribution are greater than the first quartile?
(a) 25%
(b) 50%
(c) 75%

2.20 To make a boxplot of a distribution, you must know
(a) all the individual observations.
(b) the mean and the standard deviation.
(c) the five-number summary.

2.21 The standard deviation of the 10 weights in Exercise 2.15 (use your calculator) is about LINEMEN
(a) 28.2.
(b) 28.6.
(c) 29.0.

2.22 What are all the values that a standard deviation s can possibly take?
(a) $0 \le s$
(b) $0 \le s \le 1$
(c) $-1 \le s \le 1$

2.23 The correct units for the standard deviation in Exercise 2.21 are
(a) no units—it's just a number.
(b) pounds.
(c) pounds squared.

2.24 Which of the following is most affected if an extreme high outlier is added to your data?
(a) The median
(b) The mean
(c) The first quartile

2.25 **Incomes of college grads.** According to the U.S. Census Bureau's Current Population Survey, the mean and median 2014 income of people aged 25–34 years who had a bachelor's degree but no higher degree were \$44,167 and \$51,754, respectively.[14] Which of these numbers is the mean and which is the median? Explain your reasoning.

2.26 **Household assets.** Once every three years, the Board of Governors of the Federal Reserve System collects data on household assets and liabilities through the Survey of Consumer Finances (SCF).[15] Here are some results from the 2013 survey.

(a) Transaction accounts, which include checking, savings, and money market accounts, are the most commonly held type of financial asset. The mean value of transaction accounts per household was \$270,100, and the median value was \$94,500. What explains the differences between the two measures of center?

(b) The mean value of retirement accounts per household, which includes Individual Retirement Account (IRA) balances and certain employer-sponsored accounts, was \$99,040 but the median value was \$0. What does a median of \$0 say about the percentage of households with retirement accounts?

2.27 **University endowments.** The National Association of College and University Business Officers collects data on college endowments. In 2015, its report included the endowment values of 841 colleges and universities in the United States and Canada. When the endowment values are arranged in order, what are the locations of the median and the quartiles in this ordered list?

2.28 **Pulling wood apart.** Exercise 1.46 (page 45) gives the breaking strengths of 20 pieces of Douglas fir. WOOD

(a) Give the five-number summary of the distribution of breaking strengths.

(b) Here is a stemplot of the data rounded to the nearest hundred pounds. The stems are thousands of pounds, and the leaves are hundreds of pounds.

23	0
24	1
25	
26	5
27	
28	7
29	
30	259
31	399
32	033677
33	0237

The stemplot shows that the distribution is skewed to the left. Does the five-number summary show the skew? Remember that only a graph gives a clear picture of the shape of a distribution.

2.29 **Comparing graduation rates.** An alternative presentation to compare the graduation rates in Table 1.1 (page 22) by region of the country reports five-number summaries and uses boxplots to display the distributions. Do the boxplots fail to reveal any important information visible in the stemplots of Figure 2.5 (page 64)? Which plots make it simpler to compare the regions? Why? GRADRATE

2.30 **How much fruit do adolescent girls eat?** Figure 1.14 (page 39) is a histogram of the number of servings of fruit per day claimed by 74 seventeen-year-old girls.

(a) With a little care, you can find the median and the quartiles from the histogram. What are these numbers? How did you find them?

(b) You can also find the mean number of servings of fruit claimed per day from the histogram. First use the information in the histogram to compute the sum of the 74 observations, and then use this to compute the mean. What is the relationship between the mean and median? Is this what you expected?

(c) In general, you cannot find the exact values of the median, quartiles, or mean from the histogram. What is special about the histogram of the number of servings of fruit that allows you to do this?

2.31 **Guinea pig survival times.** Here are the survival times in days of 72 guinea pigs after they were injected with infectious bacteria in a medical experiment.[16] Survival times, whether of machines under stress or cancer patients after treatment, usually have distributions that are skewed to the right. GUINPIGS

43	45	53	56	56	57	58	66	67	73	74	79
80	80	81	81	81	82	83	83	84	88	89	91
91	92	92	97	99	99	100	100	101	102	102	102
103	104	107	108	109	113	114	118	121	123	126	128
137	138	139	144	145	147	156	162	174	178	179	184
191	198	211	214	243	249	329	380	403	511	522	598

(a) Graph the distribution and describe its main features. Does it show the expected right-skew?

(b) Which numerical summary would you choose for these data? Calculate your chosen summary. How does it reflect the skewness of the distribution?

2.32 **Maternal age at childbirth.** How old are women when they have their first child? Here is the distribution of the age of the mother for all firstborn children in the United States in 2014:[17]

Age	Count	Age	Count
10–14 years	2,769	30–34 years	326,391
15–19 years	205,747	35–39 years	114,972
20–24 years	445,523	40–44 years	23,941
25–29 years	428,762	45–49 years	2,169

Tom Merton/Getty Images

The number of firstborn children to mothers under 10 or over 50 years of age represent a negligible percentage of all first births and are not included in the table.

(a) For comparison with other years and with other countries, we prefer a histogram of the *percents* in each age class rather than the counts. Explain why.

(b) How many babies were there?

(c) Make a histogram of the distribution, using percents on the vertical scale. Using this histogram, describe the distribution of the age at which women have their first child.

(d) What are the locations of the median and quartiles in the ordered list of all maternal ages? In which age classes do the median and quartiles fall?

2.33 **More on Nintendo and laparoscopic surgery.** In Exercise 1.38 (page 42), you examined the improvement in times to complete a virtual gall bladder removal for those with and without four weeks of Nintendo Wii[TM] training. The most common methods for formal comparison of two groups use $\bar{x}$ and s to summarize the data. NINTENDO

(a) What kinds of distributions are best summarized by $\bar{x}$ and s? Do you think these summary measures are appropriate in this case?

(b) In the control group, one subject improved his/her time by 229 seconds. How much does removing this observation change $\bar{x}$ and s for the control group? You will need to compute $\bar{x}$ and s for the control group, both with and without the high outlier.

(c) Compute the median for the control group with and without the high outlier. What does this show about the resistance of the median and $\bar{x}$?

2.34 **Making resistance visible.** In the *Mean and Median* applet, place three observations on the line by clicking below it: two close together near the center of the line and one somewhat to the right of these two.

(a) Pull the single rightmost observation out to the right. (Place the cursor on the point, hold down a mouse button, and drag the point.) How does the mean behave? How does the median behave? Explain briefly why each measure acts as it does.

(b) Now drag the single rightmost point to the left as far as you can. What happens to the mean? What happens to the median as you drag this point past the other two? (Watch carefully.)

2.35 **Behavior of the median.** Place five observations on the line in the *Mean and Median* applet by clicking below it.

(a) Add one additional observation *without changing the median*. Where is your new point?

(b) Use the applet to convince yourself that when you add yet another observation (there are now seven in all), the median does not change, no matter where you put the seventh point. Explain why this must be true.

2.36 **Never on Sunday: Also in Canada?** Exercise 1.5 (page 20) gives the number of births in the United States on each day of the week during an entire year. The boxplots in Figure 2.6 are based on more detailed data from Toronto, Canada: the number of births on each of the 365 days in a year, grouped by day of the week.[18] Based on these plots, compare the day-of-the-week distributions using shape, center, and variability. Summarize your findings.

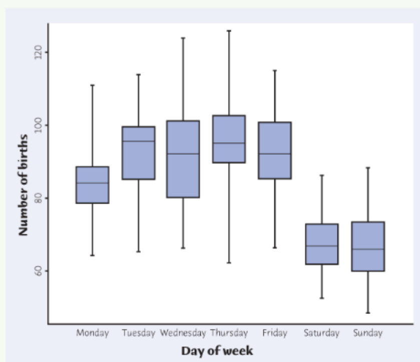

Figure 2.6

Moore/Notz/Fligner, *The Basic Practice of Statistics*, 8e, © 2018 W.H. Freeman and Company

FIGURE 2.6

Boxplots of the distributions of numbers of births in Toronto, Canada, on each day of the week during a year, for Exercise 2.36.

2.37 **Thinking about means.** Table 1.2 (page 24) gives the percent of minority residents in each of the states. For the nation as a whole, 42.8% of residents are minorities. Find the mean of the 51 entries in Table 1.2. It is *not* 42.8%. Explain carefully why this happens. (*Hint:* The states with the largest populations are California, Texas, New York, and Florida. Look at their entries in Table 1.2.) MINORITY

2.38 **Thinking about means and medians.** In 2014, approximately 2% of hourly rate workers were being paid at the federal minimum wage level. Would federal legislation to increase the minimum wage have a greater effect on the mean or the median income of *all* workers? Explain your answer.

2.39 **A standard deviation contest.** This is a standard deviation contest. You must choose four numbers from the whole numbers 0 to 10, with repeats allowed.

(a) Choose four numbers that have the smallest possible standard deviation.

(b) Choose four numbers that have the largest possible standard deviation.

(c) Is more than one choice possible in either part (a) or (b)? Explain.

2.40 **You create the data.** Create a set of seven numbers (repeats allowed) that have the five-number summary

$$\text{Minimum} = 4 \quad Q_1 = 8 \quad M = 12 \quad Q_3 = 15 \quad \text{Maximum} = 19$$

paid at the federal minimum wage level. Would federal legislation to increase the minimum wage have a greater effect on the mean or the median income of *all* workers? Explain your answer.

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(c) Is more than one choice possible in either part (a) or (b)? Explain.

2.40 **You create the data.** Create a set of seven numbers (repeats allowed) that have the five-number summary

$$\text{Minimum} = 4 \quad Q_1 = 8 \quad M = 12 \quad Q_3 = 15 \quad \text{Maximum} = 19$$

There is more than one set of seven numbers with this five-number summary. What must be true about the seven numbers to have this five-number summary?

2.41 **You create the data.** Give an example of a small set of data for which the mean is greater than the third quartile.

2.42 **Adolescent obesity.** Adolescent obesity is a serious health risk affecting more than 5 million young people in the United States alone. Laparoscopic adjustable gastric banding has the potential to provide a safe and effective treatment. Fifty adolescents between 14 and 18 years old with a body mass index (BMI) higher than 35 were recruited from the Melbourne, Australia, community for the study.[19] Twenty-five were randomly selected to undergo gastric banding, and the remaining 25 were assigned to a super vised lifestyle intervention program involving diet, exercise, and behavior modification. All subjects were followed for two years. Here are the weight losses in kilograms for the subjects who completed the study: GASTRIC

Gastric Banding					
35.6	81.4	57.6	32.8	31.0	37.6
−36.5	−5.4	27.9	49.0	64.8	39.0
43.0	33.9	29.7	20.2	15.2	41.7
53.4	13.4	24.8	19.4	32.3	22.0

Lifestyle Intervention					
6.0	2.0	−3.0	20.6	11.6	15.5
−17.0	1.4	4.0	−4.6	15.8	34.6
6.0	−3.1	−4.3	−16.7	−1.8	−12.8

(a) In the context of this study, what do the negative values in the data set mean?

(b) Give a graphical comparison of the weight loss distribution for both groups using side-by-side boxplots. Provide appropriate numerical summaries for the two distributions, and identify any high outliers in either group. What can you say about the effects of gastric banding versus lifestyle intervention on weight loss for the subjects in this study?

(c) The measured variable was weight loss in kilograms. Would two subjects with the same weight loss always have similar benefits from a weight-reduction program? Does it depend on their initial weights? Other variables considered in this study were the percent of excess weight lost and the reduction in BMI. Do you see any advantages to either of these variables when comparing weight loss for two groups?

(d) One subject from the gastric-banding group dropped out of the study, and seven subjects from the lifestyle group dropped out. Of the seven dropouts in the lifestyle group, six had gained weight at the time they dropped out. If all subjects had completed the study, how do you think it would have affected the comparison between the two groups?

Exercises 2.43 through 2.49 ask you to analyze data without having the details outlined for you. The exercise statements give you the ***state*** *step of the four-step process. In your work, follow the* ***plan, solve,*** *and* ***conclude*** *steps as illustrated in Example 2.9 (page 64).*

Exercises 2.43 through 2.49 ask you to analyze data without having the details outlined for you. The exercise statements give you the ***state*** *step of the four-step process. In your work, follow the* ***plan, solve,*** *and* ***conclude*** *steps as illustrated in Example 2.9 (page 64).*

2.43 4step **Protective equipment and risk taking.** Studies have shown that people who are using safety equipment when engaging in an activity tend to take increased risks. Will risk taking increase when people are not aware they are wearing protective equipment and are engaged in an activity that cannot be made safer by this equipment? Participants in the study were falsely told they were taking part in an eye-tracking experiment for which they needed to wear an eye-tracking device. Eighty subjects were divided at random into two groups of 40 each, with one group wearing the tracking device mounted on a baseball cap and the other group wearing it mounted on a bicycle helmet. Subjects were told that the helmet or cap was just being used to mount the eye tracker. All subjects watched an animated balloon on a video screen and pressed a button to inflate it. The balloon was programmed to burst at a random point, but until that point, each press of the button inflated the balloon further and increased the amount of fictional currency a subject would earn. Subjects were free to stop pumping at any point and keep their earnings, knowing that if the balloon burst they would lose all earnings for that round. The score was the average number of pumps on the trials, with lower scores corresponding to less risk taking and more conservative play. Here are the first 10 observations from each group:[20] HELMET

Helmet:	3.67	36.50	29.28	30.50	24.08
	32.10	50.67	26.26	41.05	20.56
Baseball Cap:	29.38	42.50	41.57	47.77	32.45
	30.65	7.04	2.68	22.04	25.86

Tim Gamble and Ian Walker, "Wearing a bicycle helmet can increase risk taking and sensation seeking in adults," Psychological Science, 27 (2016), pp. 289–294: https://doi.org/10.1177/0956797615620784 (http://www.creativecommons.org/licenses/by/3.0/).

Compare the distributions for the two groups. How is wearing of a helmet related to the measure of risk behavior?

2.44 4step **Athletes' salaries.** The Montreal Canadiens were founded in 1909, and they are the longest continuously operating professional ice hockey team. The team has won 24 Stanley Cups, making them one of the most successful professional sports teams of the traditional four major sports of Canada and the United States. Table 2.1 (page 72) gives the salaries of the 2015–2016 roster.[21] Provide the team owner with a full description of the distribution of salaries and a brief summary of its most important features. HOCKEY

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TABLE 2.1 Salaries for the 2015–2016 Montreal Canadiens

Player	Salary	Player	Salary	Player	Salary
Jeff Petry	$7,000,000	Lars Eller	$2,500,000	Lucas Lesssio	$833,000
P. K. Suban	$7,000,000	Ben Scrivens	$2,300,000	Victor Bartley	$800,000
Carey Price	$6,500,000	Mike Brown	$1,250,000	Darren Dietz	$690,000
Andrei Markov	$6,000,000	Torrey Mitchell	$1,000,000	Joel Hanley	$667,000
Brendan Gallagher	$5,500,000	Nathan Beaulieu	$1,000,000	Sven Andrighetto	$650,000
Tomas Plekanec	$5,000,000	Bryan Flynn	$950,000	Mark Barberio	$600,000
Max Pacioretty	$4,000,000	Jacob De La Rose	$925,000	Steven Matteau	$575,000
Alexei Emelin	$3,900,000	Michael McCarron	$925,000	Greg Pateryn	$575,000
David Desharnais	$3,500,000	Paul Byron	$900,000	Michael Condon	$575,000
Tom Gilbert	$2,800,000	Daniel Carr	$892,000		
Alex Galchenyuk	$2,500,000	Phillip Danault	$833,000		

Olivier Samson Arcand/Getty Images

2.45 **Returns on stocks.** How well have stocks done over the past generation? The Wilshire 5000 index describes the average performance of all U.S. stocks. The average is weighted by the total market value of each company's stock, so think of the index as measuring the performance of the average investor. Here are the percent returns on the Wilshire 5000 index for the years from 1971–2015:[22] WILSHIRE

Year	Return	Year	Return	Year	Return
1971	17.68	1986	16.09	2001	−10.97
1972	17.98	1987	2.27	2002	−20.86
1973	−18.52	1988	17.94	2003	31.64
1974	−28.39	1989	29.17	2004	12.62
1975	38.47	1990	−6.18	2005	6.32
1976	26.59	1991	34.20	2006	15.88
1977	−2.64	1992	8.97	2007	5.73
1978	9.27	1993	11.28	2008	−37.34
1979	25.56	1994	−0.06	2009	29.42
1980	33.67	1995	36.45	2010	17.87
1981	−3.75	1996	21.21	2011	0.59

Returns on stocks. How well have stocks done over the past generation? The Wilshire 5000 index describes the average performance of all U.S. stocks. The average is weighted by the total market value of each company's stock, so think of the index as measuring the performance of the average investor. Here are the percent returns on the Wilshire 5000 index for the years from 1971–2015:[22] WILSHIRE

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1971	17.68	1986	16.09	2001	−10.97
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1973	−18.52	1988	17.94	2003	31.64
1974	−28.39	1989	29.17	2004	12.62
1975	38.47	1990	−6.18	2005	6.32
1976	26.59	1991	34.20	2006	15.88
1977	−2.64	1992	8.97	2007	5.73
1978	9.27	1993	11.28	2008	−37.34
1979	25.56	1994	−0.06	2009	29.42
1980	33.67	1995	36.45	2010	17.87
1981	−3.75	1996	21.21	2011	0.59

Year	Return	Year	Return	Year	Return
1982	18.71	1997	31.29	2012	16.12
1983	23.47	1998	23.43	2013	34.02
1984	3.05	1999	23.56	2014	12.07
1985	32.56	2000	−10.89	2015	−0.24

What can you say about the distribution of yearly returns on stocks?

2.46 **Do good smells bring good business?** Businesses know that customers often respond to background music. Do they also respond to odors? Nicolas Guéguen and his colleagues studied this question in a small pizza restaurant in France on Saturday evenings in May. On one evening, a relaxing lavender odor was spread through the restaurant; on another evening, a stimulating lemon odor; a third evening served as a control, with no odor. Table 2.2 (page 73) shows the amounts (in euros) that customers spent on each of these evenings.[23] Compare the three distributions. Were both odors associated with increased customer spending? ODORS

TABLE 2.2 Amount spent (Euros) by customers in a restaurant when exposed to odors

No Odor									
15.9	18.5	15.9	18.5	18.5	21.9	15.9	15.9	15.9	15.9
15.9	18.5	18.5	18.5	20.5	18.5	18.5	15.9	15.9	15.9
18.5	18.5	15.9	18.5	15.9	18.5	15.9	25.5	12.9	15.9
Lemon Odor									
18.5	15.9	18.5	18.5	18.5	15.9	18.5	15.9	18.5	18.5
15.9	18.5	21.5	15.9	21.9	15.9	18.5	18.5	18.5	18.5
25.9	15.9	15.9	15.9	18.5	18.5	18.5	18.5		
Lavender Odor									
21.9	18.5	22.3	21.9	18.5	24.9	18.5	22.5	21.5	21.9
21.5	18.5	25.5	18.5	18.5	21.9	18.5	18.5	24.9	21.9
25.9	21.9	18.5	18.5	22.8	18.5	21.9	20.7	21.9	22.5

2.47 **Policy justification: Pragmatic vs. moral.** How does a leader's justification of his/her organization's policy affect support for the policy? This study compared a moral, pragmatic, and ambiguous justification for three policy proposals: a politician's plan to fund a retirement planning agency, a state governor's plan to repave state highways, and a president's plan to outlaw child labor in a developing country. For example, for the retirement agency proposal, the moral justification was the importance of retirees "to live with dignity and comfort," the pragmatic was "to not drain public funds," and the ambiguous was "to have sufficient funds." Three hundred seventy-four volunteer subjects were assigned at random to read all three proposals: 122 subjects read the three proposals with a moral justification, 126 subjects read each with a pragmatic justification, and 126 subjects with an ambiguous justification. Several questions measuring support for each policy proposal were answered by the subject to create a support score for each proposal, and their scores for the three proposals were then averaged to create an index of policy support, higher values indicating greater support.[24] Here are the first five observations: JUSTIFY

Justification:	Pragmatic	Ambiguous	Pragmatic	Moral	Ambiguous
Policy support index:	5	7	4.75	7	5.75

The first individual read the proposals with a pragmatic justification with a policy support index of 5, the second with an ambiguous justification and a policy support index of 7, and so forth. Compare the three distributions. How does the support index vary with the type of justification?

2.48 **Does playing video games improve surgical skill?** In laparoscopic surgery, a video camera and several thin instruments are inserted into the patient's abdominal cavity. The surgeon uses the image from the video camera positioned inside the patient's body to perform the procedure by manipulating the instruments that have been inserted. The Top Gun Laparoscopic Skills and Suturing Program was developed to help surgeons develop the skill set necessary for laparoscopic surgery. Because of the similarity in many of the skills involved in video games and laparoscopic surgery, it was hypothesized that surgeons with greater prior video game experience might acquire the skills required in laparoscopic surgery more easily. Thirty-three surgeons participated in the study and were classified into the three categories—never used, under three hours, and more than three hours—depending on the number of hours they played video games at the height of their video game use. They also performed Top Gun drills and received a score based on the time to complete the drill and the number of errors made, with lower scores indicating better performance. Here are the Top Gun scores and video game categories for the 33 participants:[25] TOPGUN

Never played:	9379	8302	5489	5334	4605	4789	9185	7216	9930
	4828	5655	4623	7778	8837	5947			
Under three hours:	5540	6259	5163	6149	4398	3968	7367	4217	5716
Three or more hours:	7288	4010	4859	4432	4845	5394	2703	5797	3758

Compare the distributions for the three groups. How is prior video game experience related to Top Gun scores?

2.49 **Cholesterol levels and age.** The National Health and Nutrition Examination Survey (NHANES) is a unique survey that combines interviews and physical examinations.[26] It includes basic demographic information; questions about topics such as diet, physical activity, and prescription medications; as well as the results of a physical examination measuring a variety of variables, including blood pressure and cholesterol levels. The program began in the early 1960s, and the survey currently examines a nationally representative sample of about 5000 persons each year. You will work with the total cholesterol measurements (mg/dL) obtained from participants in the survey in 2009–2010. CHOLEST

To examine changes in cholesterol with age, we consider only the 3044 participants between 20 and 50 years of age and have classified them into the three age categories 20s, 30s, and 40s. The full data set is too large to print here, but here are the first 10 individuals:

Age category:	30s	20s	20s	40s	30s	40s	20s	30s	30s	20s
Total cholesterol:	135	160	299	197	196	202	175	216	181	149

The first individual is in the 30s with a total cholesterol of 135, the second in the 20s with total cholesterol of 160, and so forth.

(a) Use graphical and numerical summaries to compare the three distributions. How does cholesterol change with age?

(b) The ideal range of total cholesterol is below 200 mg/dL. For individuals with elevated cholesterol levels, prescription drugs are often recommended to lower levels. Among the 3044 participants between 20 and 50 years of age, 4 individuals in their 20s, 24 individuals in their 30s, and 117 individuals in their 40s were taking prescription medications to reduce their cholesterol levels. How do you think your comparison of the distribution would be changed if none of the individuals were taking medication? Explain.

Exercises 2.50 through 2.53 make use of the optional material on the 1.5 × IQR rule for suspected outliers.

2.50 **The changing face of America.** Figure 1.10 (page 30) gives a stemplot of the percent of minority residents aged 18–34 in each of the 50 states and the District of Columbia. These data are given in Table 1.2. MINORITY

(a) Give the five-number summary of this distribution.

(b) Although there do not appear to be any outliers in Figure 1.10, when you split the stems for the data in Exercise 1.10, Texas, Alabama, California, and Hawaii are separated from the remaining states. Are these four states outliers or just the largest observations in a strongly skewed distribution? What does the 1.5 × *IQR* rule say?

2.51 **Shared pain and bonding.** In Exercise 2.6, you should have noticed some low outliers in the pain group. BONDING

(a) Compute the mean and the median of the bonding scores for the pain group, both with and without the two smallest scores. Do they have more of an effect on the mean or the median? Explain why.

(b) Does the 1.5 × *IQR* rule identify these two low bonding scores as suspected outliers?

(c) Unusual observations are not necessarily mistakes. Suppose a small percentage of subjects would experience little bonding regardless of whether they were in the no-pain or the pain group. Explain how the randomization of the students to the two groups could have led to these "outliers."

2.52 **The Fortune Global 500.** The *Fortune* Global 500, also known as the Global 500, is an annual ranking of the top 500 corporations worldwide as measured by revenue. In total, the Global 500 generated $31.2 trillion in revenues in 2014. Table 2.3 provides a list of the 30 companies with the highest revenues (in billions of dollars) in 2014.[27] A stemplot or histogram shows that the distribution is strongly skewed to the right. GLOBE500

TABLE 2.3 Revenues for the top Global 500 companies in 2014

Company Name	Revenues ($b)	Company Name	Revenues ($b)
Wal-Mart Stores	485.7	McKesson	181.2
Sinopec Group	446.8	Daimler	172.3
Royal Dutch Shell	431.3	Industrial & Commercial Bank of China	163.2
China National Petroleum	428.6	Exor Group	162.1
ExxonMobil	382.6	AXA	161.2
BP	358.7	General Motors	155.9

BONDING

(a) Compute the mean and the median of the bonding scores for the pain group, both with and without the two smallest scores. Do they have more of an effect on the mean or the median? Explain why.

(b) Does the 1.5 × *IQR* rule identify these two low bonding scores as suspected outliers?

(c) Unusual observations are not necessarily mistakes. Suppose a small percentage of subjects would experience little bonding regardless of whether they were in the no-pain or the pain group. Explain how the randomization of the students to the two groups could have led to these "outliers."

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ExxonMobil	382.6	AXA	161.2
BP	358.7	General Motors	155.9
State Grid	339.4	E.On	151.5
Volkswagen	268.6	Phillips 66	149.4
Toyota Motor	247.7	General Electric	148.3
Glencore	221.0	ENI	147.2
Total	212.0	Gazprom	144.4
Chevron	203.8	Ford Motor	144.1
Samsung Electronics	195.8	Petrobras	143.7
Berkshire Hathaway	194.7	China Construction Bank	139.9
Apple	182.8	CVS Health	139.4

(a) Give the five-number summary. Explain why this summary suggests that the distribution is right-skewed.

(b) Which companies are outliers according to the 1.5 × *IQR* rule? Make a stemplot of the data. Do you agree with the rule's suggestions about which companies are and are not outliers?

(c) If you consider *all* 500 companies, the 30 companies in Table 2.3 each represent a high outlier among all Global 500 companies. Is there a common feature shared by many of the 30 companies in the table? What proportion of the total of the Global 500 revenues is accounted for by these 30 companies?

2.53 **Cholesterol for people in their 20s.** Exercise 2.49 contains the cholesterol levels of individuals in their 20s from the NHANES survey in 2009–2010. The cholesterol levels are right-skewed, with a few large cholesterol levels. Which cholesterol levels are suspected outliers by the 1.5 × *IQR* rule? CHOLES20

[*] This short section is optional.